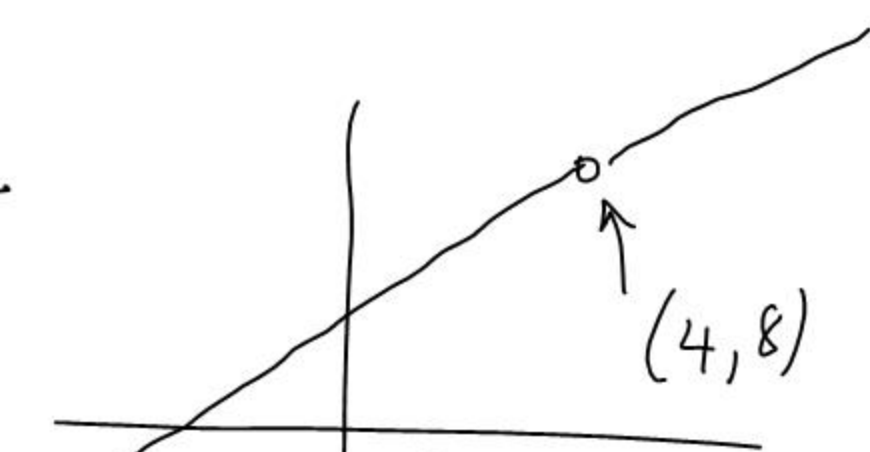


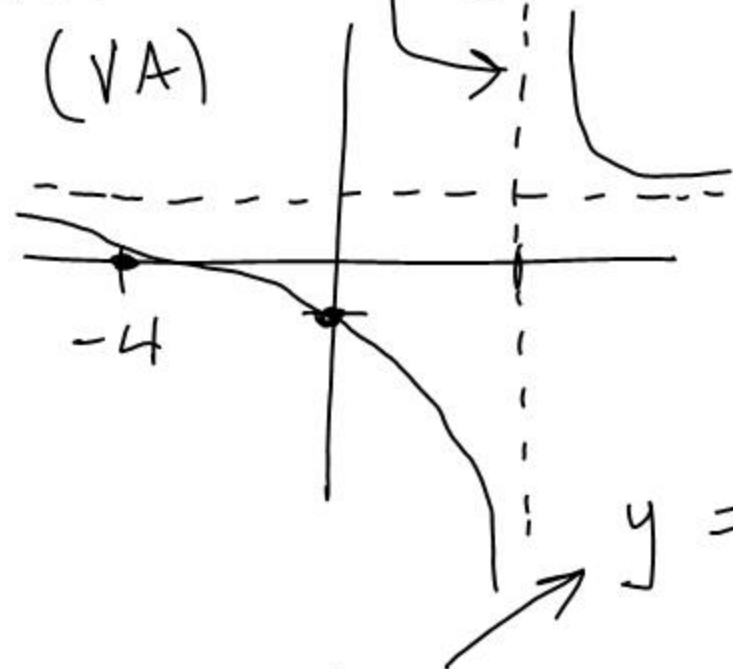
$$y = \frac{x^2 - 16}{x - 4}$$



(4, 8)

Essential
Discontinuity
(VA)

Removable
Discontinuity



$$y = \frac{x + 4}{x - 4}$$

$$y = \lim_{x \rightarrow \infty} \frac{x + 4}{x - 4} = 1$$

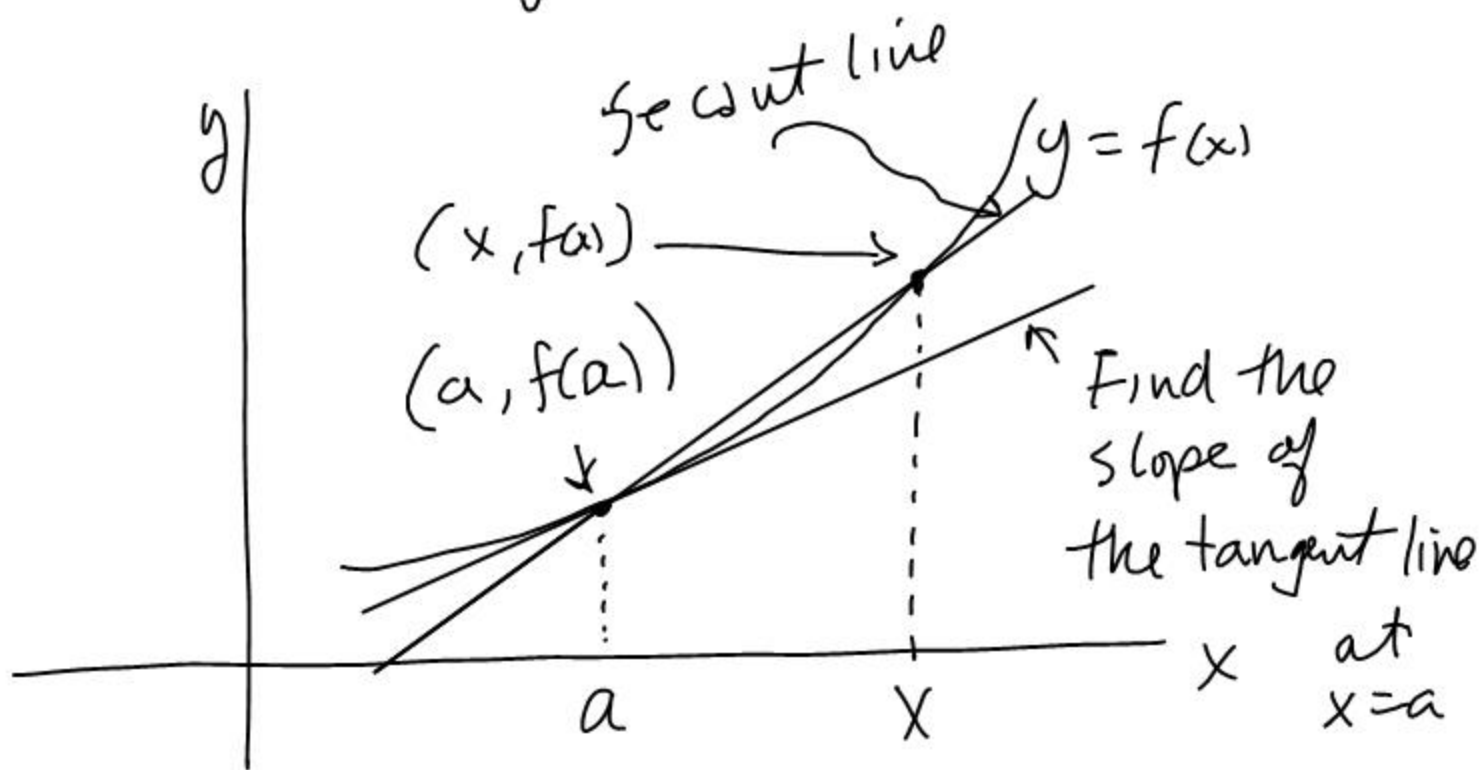
Horizontal
Asymptote

$$x = 1004 : \frac{1008}{1000} = 1.008$$

EX. $\lim_{x \rightarrow \infty} \frac{x + 2}{x^2 + 2} = 0 \rightarrow$ H.A. is $y = 0$

$$x = 1000 : \frac{1002}{1000002} \approx \frac{1}{1000}$$

The Tangent Line Problem



secant line slope: $\frac{f(x) - f(a)}{x - a}$

tangent line slope:

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

The derivative of $f(x)$
at $x=a$.

$$f(2) = 2^3$$

Ex Find $f'(2)$ for $f(x) = x^3$.

$$f'(2) = \lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{x-2}}$$

$$= 2^2 + 2(2) + 4$$

$$= 12$$

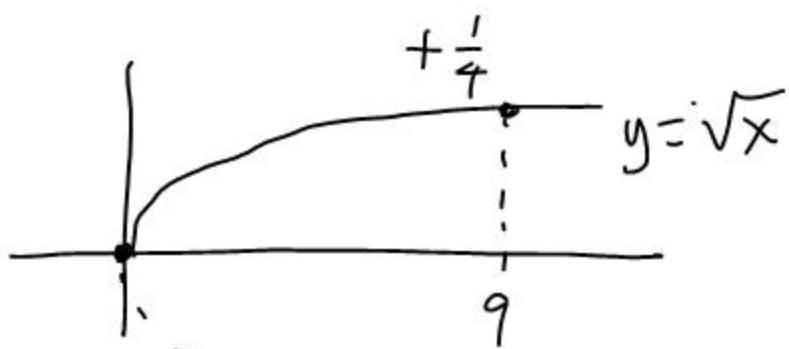
Sum of cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

diff of cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Ex. Find $f'(9)$ for $f(x) = \sqrt{x}$



$$f'(9) = \lim_{x \rightarrow 9} \frac{\sqrt{x} - \sqrt{9}}{x - 9} \cdot \frac{\sqrt{x} + \sqrt{9}}{\sqrt{x} + \sqrt{9}}$$

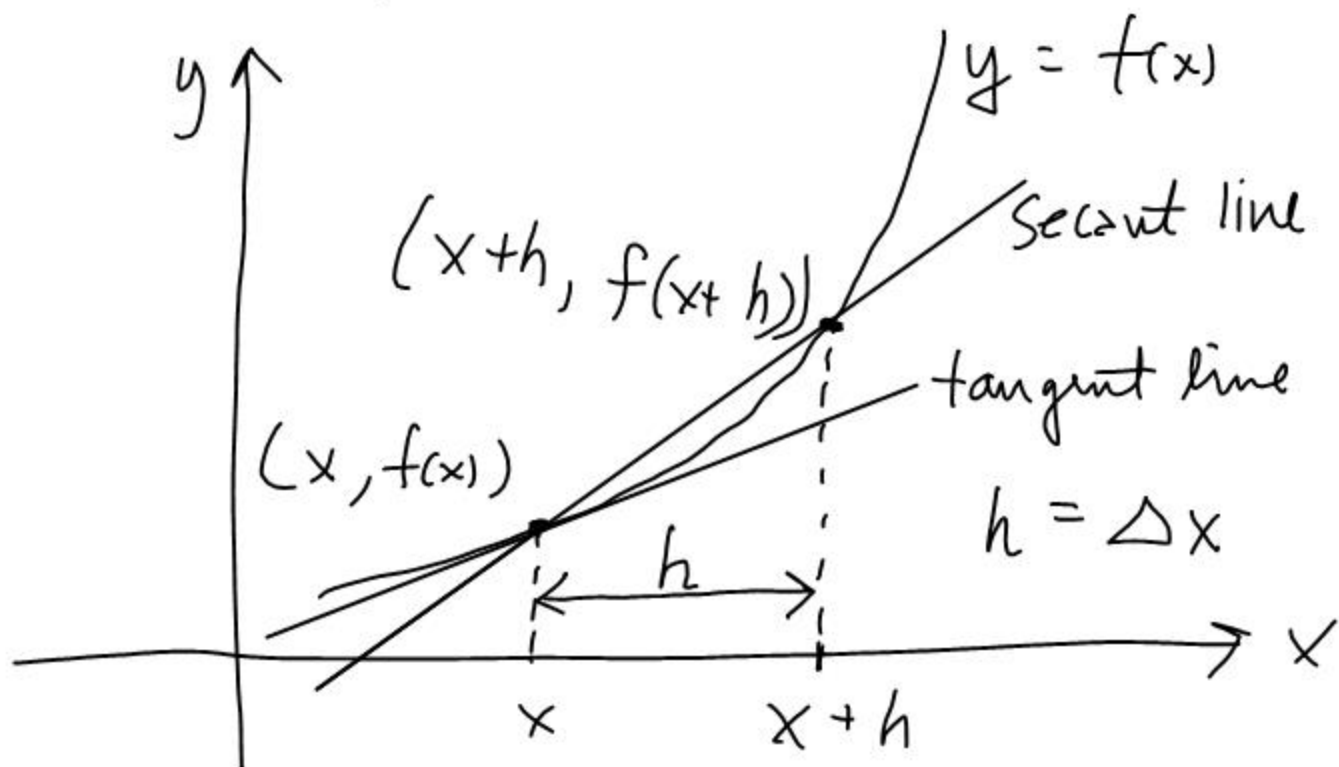
$$= \lim_{x \rightarrow 9} \frac{\cancel{x - 9} \cdot 1}{(\cancel{x - 9})(\sqrt{x} + 3)} = \frac{1}{6}$$

← The principal square root
 $\sqrt{9} = 3$

$$\sqrt{9} \neq -3$$

$$\sqrt[3]{8} = 2$$

The tangent problem (part 2)



The tangent line slope at x is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$(x+h) - x$$

The derivative function for f .

$$f(x+h) = \sqrt{4-(x+h)}$$

Ex. Find $f'(x)$ for $f(x) = \sqrt{4-x}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{4-(x+h)} - \sqrt{4-x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{4-x-h} - \sqrt{4-x}}{h} \cdot \frac{\sqrt{4-x-h} + \sqrt{4-x}}{\sqrt{4-x-h} + \sqrt{4-x}}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{4-x-h} - \cancel{(4-x)}}{\cancel{(\sqrt{4-x-h} + \sqrt{4-x})}}$$

$$= \frac{-1}{\sqrt{4-x} + \sqrt{4-x}} = \frac{-1}{2\sqrt{4-x}}$$

$$f'(0) = \frac{-1}{2\sqrt{4-0}} = \frac{-1}{4}$$

Exercises

(1) Find $f'(3)$ for $f(x) = x^2 - x$
 $x \rightarrow a$

(2) Find $f'(2)$ for $f(x) = \sqrt{x+7}$

(3) Find $f'(x)$ for $f(x) = x^2$

(4) Find $f'(x)$ for $f(x) = \frac{1}{\sqrt{x}}$
 $h \rightarrow 0$