$$z = a + bi$$

$$\bar{z} = z^* = a - bi \quad \text{(conjugate)}$$

Limits

$$f(x) = \frac{x^2 - 1}{x - 1}$$

x	$f(x)$
0.9	1.9
0.99	1.99
0.999	1.999
1.1	2.1
1.01	2.01
1.0001	2.0001

$x \to 1^-$ $y \to 2$

$x \to 1^+$ $y \to 2$

$$\boxed{\lim_{x \to 1^-} f(x) = 2}$$

one-sided limits

$$\boxed{\lim_{x \to 1^+} f(x) = 2}$$

2-sided limit

$$\boxed{\lim_{x \to 1} f(x) = 2}$$

the two 1-sided limits agree

$$\frac{-0.1}{0.1}$$

Ex. $\displaystyle\lim_{x\to 3^-} \frac{x-3}{|x-3|} = -1$

$\displaystyle\lim_{x\to 3^+} \frac{x-3}{|x-3|} = 1$

$$\frac{0.1}{0.1}$$

$\displaystyle\lim_{x\to 3} \frac{x-3}{|x-3|}$ dne (1-sided limits don't agree)

Ex. $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} = 0.1\overline{6} = \frac{1}{6}$

2.000001

~~The factoring trick~~

Ex $\displaystyle\lim_{x\to -1} \frac{x^2+3x+2}{x^2-2x-3} = \lim_{x\to -1} \frac{(x+2)(x+1)}{(x-3)(x+1)}$

$= \dfrac{-1+2}{-1-3} = -\dfrac{1}{4}$

same as $\dfrac{x+2}{x-3}$

<u>except</u> for $x=-1$

Ex. $\lim_{x \to 2} \frac{\sqrt{x+7} - 3}{x-2} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3}$

$= \lim_{x \to 2} \frac{\cancel{x+7-9}^{\,1}}{\cancel{(x-2)}(\sqrt{x+7}+3)} = \frac{1}{6}$

The conjugate trick

Do obvious algebra trick

Ex. $\lim_{x \to 5} \frac{\frac{x+2}{x-1} - \frac{7}{4}}{x-5}$

$= \lim_{x \to 5} \frac{\frac{(x+2)(4)}{(x-1)(4)} - \frac{7(x-1)}{4(x-1)}}{\cancel{x-5}/1} \cdot \frac{1}{x-5}$

$= \lim_{x \to 5} \frac{4x+8-7x+7}{4(x-1)(x-5)}$

$= \lim_{x \to 5} \frac{-3x+15}{4(x-1)(x-5)} = \lim_{x \to 5} \frac{-3\cancel{(x-5)}}{4(x-1)\cancel{(x-5)}}$

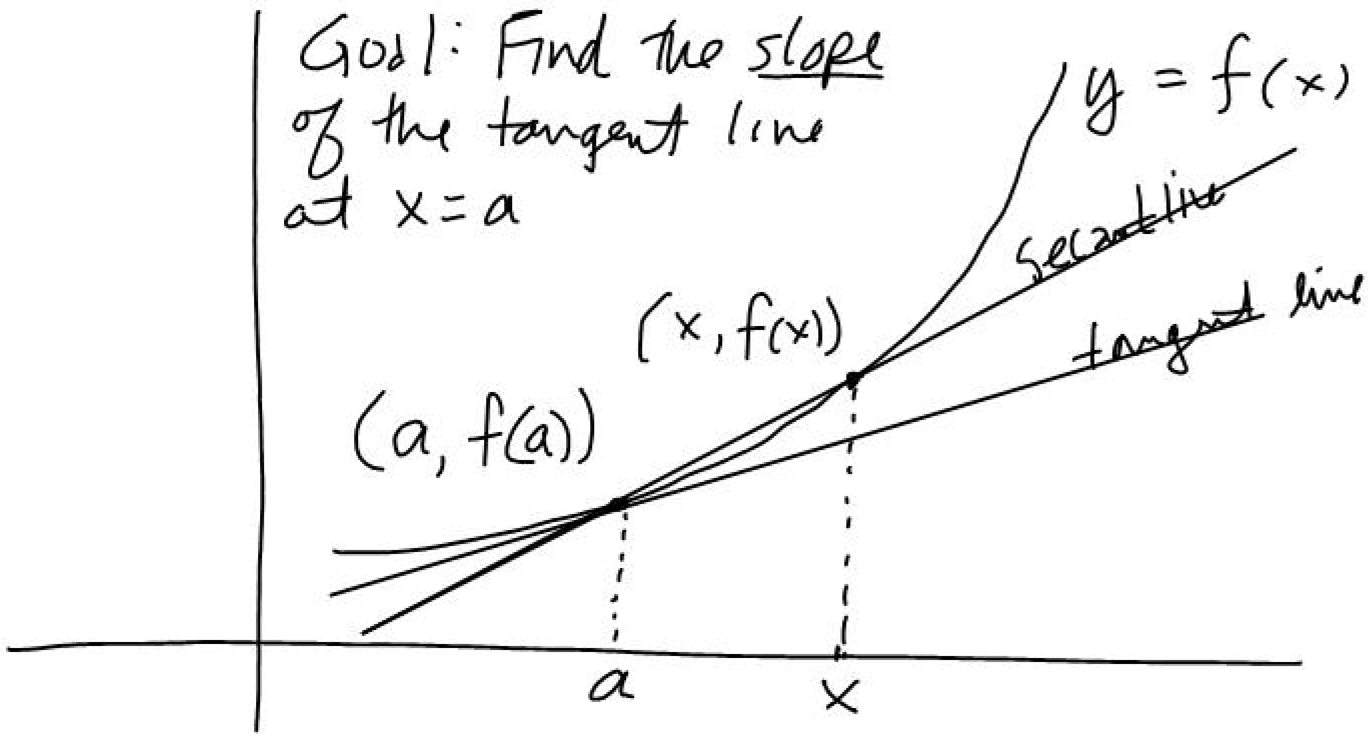
$= \frac{-3}{16}$

Ex. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

Need more limit tricks

Why limits?

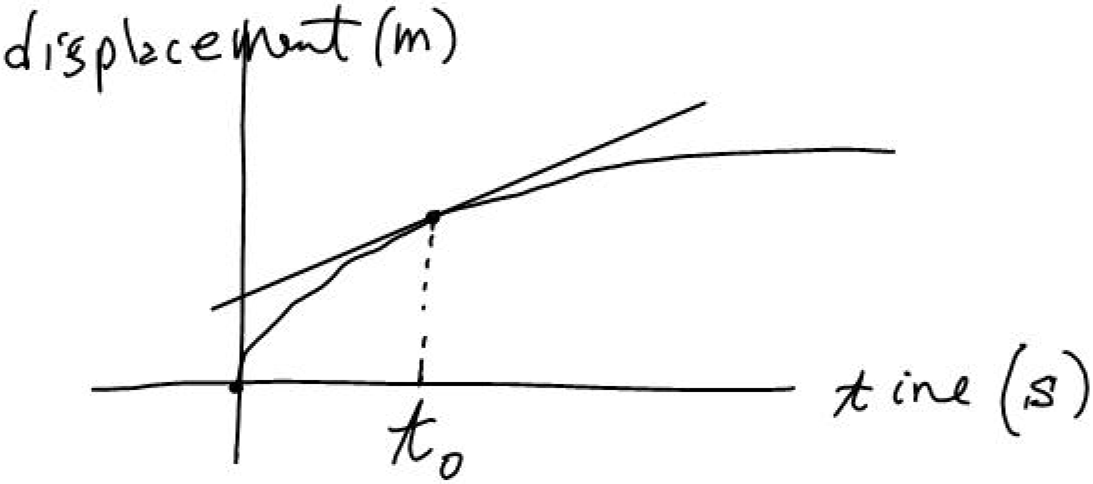
Goal: Find the slope of the tangent line at $x = a$



secant line slope: $\dfrac{f(a) - f(x)}{a - x}$

tangent line slope:

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(a) - f(x)}{a - x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$



$$f'(t_0) = \frac{\Delta \text{ displacement}}{\Delta \text{ time}} = \text{velocity}$$

$$\frac{m}{s}$$

Calculus

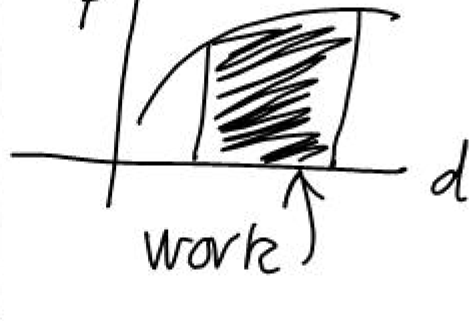
Differential

- rates of change

(tangent line problems)

Integral

- accumulated rates of change



Series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$