#6 $f(x-2) = (x-2)^3 + 12(x-2)^2 + 6(x-2) + 3$

$x^2 - 4x + 4$

$x^3 - 6x^2 + 12x - 8 + 12x^2 - 48x + 48$

$+6x - 12 + 3$

$x^3 + 6x^2 - 30x + 31$

#8

$f(10) = a \cdot 10^4 + b \cdot 10^3 + c \cdot 10^2 + d \cdot 10 + 4$

$= 32584$

$= 3 \times 10000 + 2 \times 1000 + 5 \times 100 + 8 \times 10 + 4$

#4 $f(x) \cdot g(x) = (x^3 + ax^2 - x + 2)(2x^2 + bx + c)$

$= 2x^5 + bx^4 + cx^3$

$2ax^4 + abx^3 + acx^2$

$-2x^3 - bx^2 - cx$

$4x^2 + 2bx + 2c$

$(2b - c)x + 2c$

$2x^5 + (2a+b)x^4 + (ab + c - 2)x^3 + (ac - b + 4)x^2$

$2a^{-3}$

$a^3 + a^2 - 2$

$a^2 + a^3 + 4$

$2b - a^2$

$= -5 \rightarrow a = -1$

$= 3$

$= 5$

$= -8 \rightarrow b = -3$

$c = 2$

Division

$$x^2 + 2x - 1 \overline{)\, x^4 - 2x^3 + x^2 - x - 1}$$

$x^2 - 4x + 10 - \dfrac{25x - 9}{x^2 + 2x - 1}$

$-x^4 \mp 2x^3 + x^2$

$-4x^3 + 2x^2 - x$

$+4x^3 + 8x^2 \mp 4x$

$10x^2 - 5x - 1$

$-10x^2 \mp 20x + 10$

$-25x + 9$

If you graph $y = \dfrac{x^4 - 2x^3 + x^2 - x - 1}{x^2 + 2x - 1}$

what are the asymptotes?

- **vertical asymptotes**

$$x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 4}}{2}$$

2 vertical asymptotes $\boxed{x = -1 \pm \sqrt{2}}$

- **non-vertical asymptotes** (NVA)

$$y = x^2 - 4x + 10$$

The NVA describes what happens as $x \to \pm\infty$. Every rational function has exactly one NVA.

Factoring Polynomials

Ex. Factor $x^3 + 4x^2 - 2x - 20$

Guess + check: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$1^3 + 4(1)^2 - 2(1) - 20 = -17$

$(-1)^3 + 4(-1)^2 - 2(-1) - 20 = -15$

$2^3 + 4(2)^2 - 2(2) - 20 = 0$ ✓

2 is a zero $\Rightarrow$ $(x-2)$ is a factor

$$
\begin{array}{r}
x^2 + 6x + 10 \\
x - 2 \overline{)\, x^3 + 4x^2 - 2x - 20} \\
\underline{-x^3 + 2x^2} \\
6x^2 - 2x \\
\underline{-6x^2 + 12x} \\
10x - 20 \\
\underline{-10x + 20} \\
0
\end{array}
$$

$$(x-2)(x^2+6x+10)$$

$$\Delta = 6^2 - 4(1)(10) = -4$$

Ex. Factor $x^3 + 4x^2 - 2x - 20$

1	1	4	−2	−20
		1	5	3
	1	5	3	−17 = f(1)

2	1	4	−2	−20
		2	12	20
	1	6	10	0 = f(2)

$$(x-2)(x^2+6x+10)$$

EX Factor

$$x^4 - 5x^3 - 2x^2 + 46x - 60$$

2	1	−5	−2	46	−60
		2	−6	−16	60
	1	−3	−8	30	0

$x^3 - 3x^2 - 8x + 30$

−3	1	−3	−8	30
		−3	18	−30
	1	−6	10	0

$(x-2)(x+3)(x^2-6x+10)$

$x = 2$ $\quad$ $x = -3$

Descartes' Rule

How many positive and negative roots are possible?

$+x^4 - 5x^3 - 2x^2 + 46x - 60$

3 or 1 positive roots

$f(-x) = +x^4 + 5x^3 - 2x^2 - 46x - 60$

1 neg. (real) root

Ex. $x^3 + 2x^2 + x + 5 = 0$

No pos. roots

HW 3I #1 bc, 2 bc

3J #1a, #2

Quiz on last lessons