prob and stats - November 16, 2017 *[179 marks]*

#1   The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{x^2}{14},\ x \in \{1, 2, k\},\ \text{where}\ k > 0$$

.
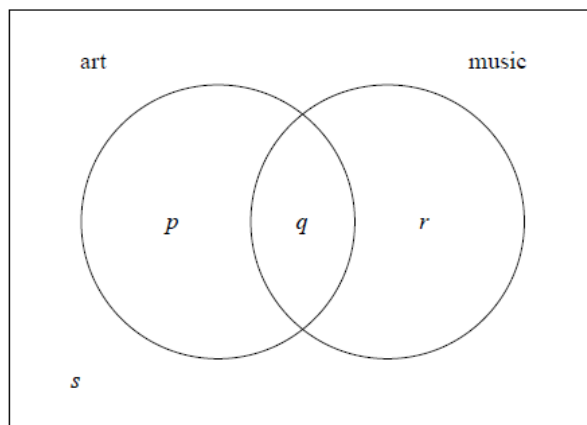
1a. Write down
$P(X = 2)$.   *[1 mark]*

1b. Show that
$k = 3$.   *[4 marks]*

1c. Find
$E(X)$.   *[2 marks]*

#2   In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p , q , r and s represent numbers of students.
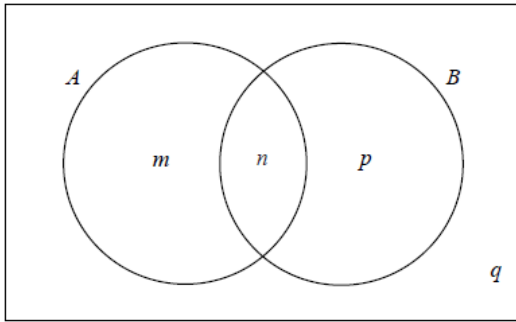

2a. (i)   Write down the value of s .   *[5 marks]*

(ii)   Find the value of q .

(iii) Write down the value of p and of r .

2b. (i)   A student is selected at random. Given that the student takes music, write down the probability the student   *[4 marks]*
takes art.

(ii)   **Hence**, show that taking music and taking art are **not** independent events.

2c. Two students are selected at random, one after the other. Find the probability that the first student takes **only**   *[4 marks]*
music and the second student takes **only** art.

#3 The Venn diagram below shows events A and B where
$P(A) = 0.3$,
$P(A \cup B) = 0.6$ and
$P(A \cap B) = 0.1$. The values m, n, p and q are probabilities.

3a. (i) Write down the value of n. [4 marks]

(ii) Find the value of m, of p, and of q.

3b. Find [2 marks]
$P(B')$.

#4 A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.
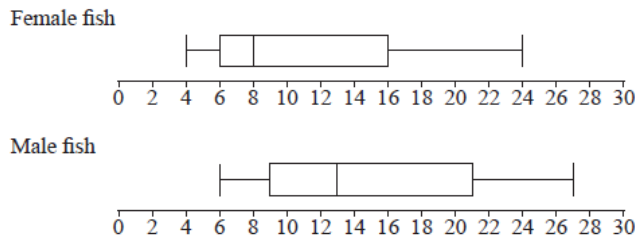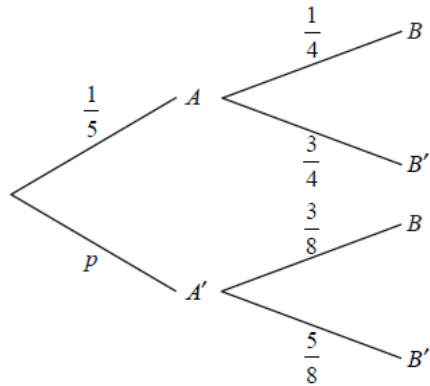

4a. Find the range of the lengths of **all** 200 fish. [3 marks]

4b. Four cumulative frequency graphs are shown below. [2 marks]
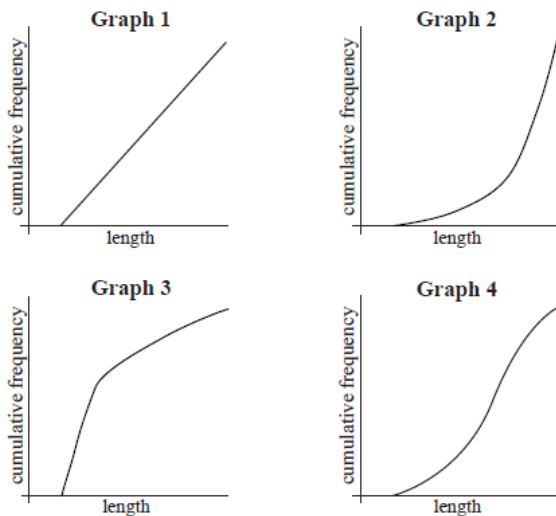
Which graph is the best representation of the lengths of the **female** fish?

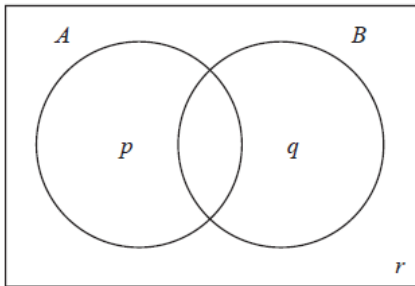**#5** The diagram below shows the probabilities for events A and B , with $P(A') = p$.

5a. Write down the value of p .   *[1 mark]*

5b. Find
$P(B)$.   *[3 marks]*

5c. Find
$P(A'|B)$.   *[3 marks]*

#6   Consider the events A and B, where
$P(A) = 0.5$,
$P(B) = 0.7$ and
$P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.

6a. Write down the value of   *[3 marks]*

(i) p ;

(ii) q ;

(iii) r.

6b. Find the value of   *[2 marks]*
$P(A|B')$.

6c. Hence, or otherwise, show that the events A and B are **not** independent.   *[1 mark]*

#7

Let *X* be normally distributed with mean 100 cm and standard deviation 5 cm.

7a. On the diagram below, shade the region representing
$P(X > 105)$.

[2 marks]
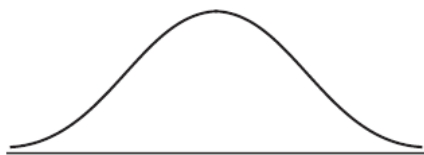
7b. Given that
$P(X < d) = P(X > 105)$, find the value of
d .

[2 marks]

7c. Given that
$P(X > 105) = 0.16$ (correct to two significant figures), find
$P(d < X < 105)$.

[2 marks]

#8

The letters of the word PROBABILITY are written on 11 cards as shown below.

| P | R | O | B | A | B | I | L | I | T | Y |

Two cards are drawn at random without replacement.

Let *A* be the event the first card drawn is the letter A.

Let *B* be the event the second card drawn is the letter B.

8a. Find
$P(A)$.

[1 mark]

8b. Find
$P(B|A)$.

[2 marks]

8c. Find
$P(A \cap B)$.

[3 marks]

#9

Let *A* and *B* be independent events, where
$P(A) = 0.6$ and
$P(B) = x$.

9a. Write down an expression for
$P(A \cap B)$.

[1 mark]

9b. Given that
$P(A \cup B) = 0.8$,

(i)   find *x* ;

(ii)   find
$P(A \cap B)$.

[4 marks]

9c. **Hence**, explain why *A* and *B* are **not** mutually exclusive.

[1 mark]

#10

A box contains 100 cards. Each card has a number between one and six written on it.The following table shows the frequencies for each number.

Number	1	2	3	4	5	6
Frequency	26	10	20	k	29	11

10a. Calculate the value of k.   *[2 marks]*

10b. Find   *[5 marks]*

   (i)   the median;

   (ii) the interquartile range.

#11

There are 20 students in a classroom. Each student plays only one sport. The tablebelow gives their sport and gender.

	Football	Tennis	Hockey
Female	5	3	3
Male	4	2	3

11a. One student is selected at random.   *[4 marks]*

   (i)   Calculate the probability that the student is a male or is a tennis player.

   (ii) Given that the student selected is female, calculate the probability that thestudent does not play football.

11b. Two students are selected at random. Calculate the probability that neitherstudent plays football.   *[3 marks]*

#12

There are nine books on a shelf. For each book,x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.
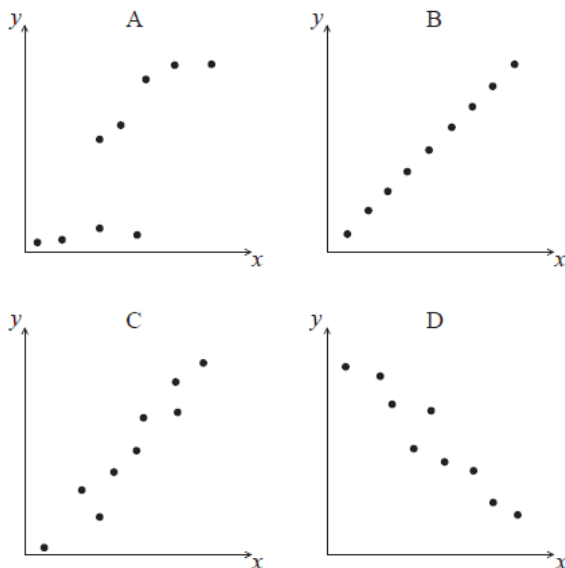
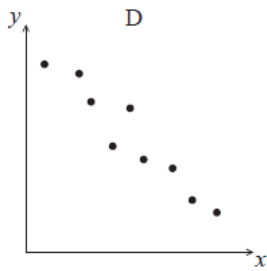12a. Write down the possible minimum and maximum values of r .   *[2 marks]*

12b. Given that   *[1 mark]*
$r = 0.95$, which of the following diagrams best represents the data.

12c. For the data in diagram D , which**two** of the following expressions describethe correlation between *x* and *y* ?   *[2 marks]*

perfect, zero, linear, strong positive, strong negative, weak positive, weak negative

#13
A data set has a mean of 20 and a standard deviation of 6.

13a. Each value in the data set has 10 added to it. Write down the value of   *[2 marks]*

(i)   the new mean;

(ii)   the new standard deviation.

13b. Each value in the original data set is multiplied by 10.   *[3 marks]*

(i)   Write down the value of the new mean.

(ii)   Find the value of the new variance.

#14
The following box-and-whisker plot represents the examination scores of a group of students.

Examination scores
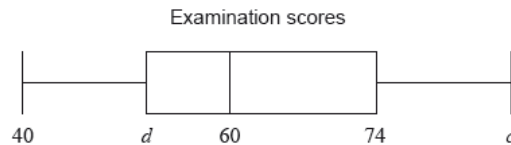
14a. Write down the median score.   *[1 mark]*

The range of the scores is 47 marks, and the interquartile range is 22 marks.

14b. Find the value of   *[4 marks]*

(i) c;

(ii) d.

#15
A discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X=x)$	$\dfrac{3}{10}$	$\dfrac{4}{10}$	$\dfrac{2}{10}$	p

15a. Find p.   *[3 marks]*

15b. Find $\mathrm{E}(X)$.   *[3 marks]*

#16

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.
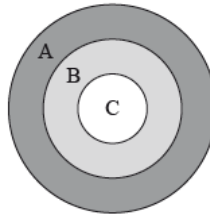
Ann's 1st roll	Bob's 1st roll	Ann's 2nd roll	Bob's 2nd roll	Ann's 3rd roll
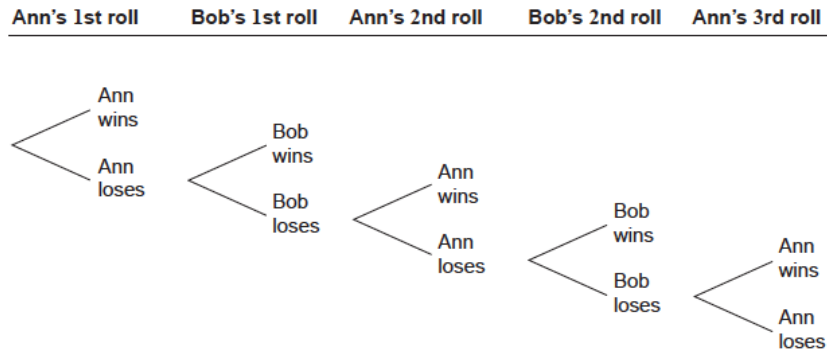
16a. Find the probability that Ann wins on her first roll. *[2 marks]*

16b. Find the probability that Ann wins the game. *[7 marks]*

The following diagram shows a board which is divided into three regions A, B and C.

#17

A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\dfrac{5}{20}$	$\dfrac{4}{20}$	$\dfrac{1}{20}$

17a. Find the probability that the dart does **not** hit the board. *[3 marks]*

17b. The contestant scores points as shown in the following table. *[4 marks]*

Region	A	B	C	Does not hit the board
Points	0	q	10	−3

Given that the game is fair, find the value of q.

18. Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. *[6 marks]*

The taxicabs are randomly assigned by the company.
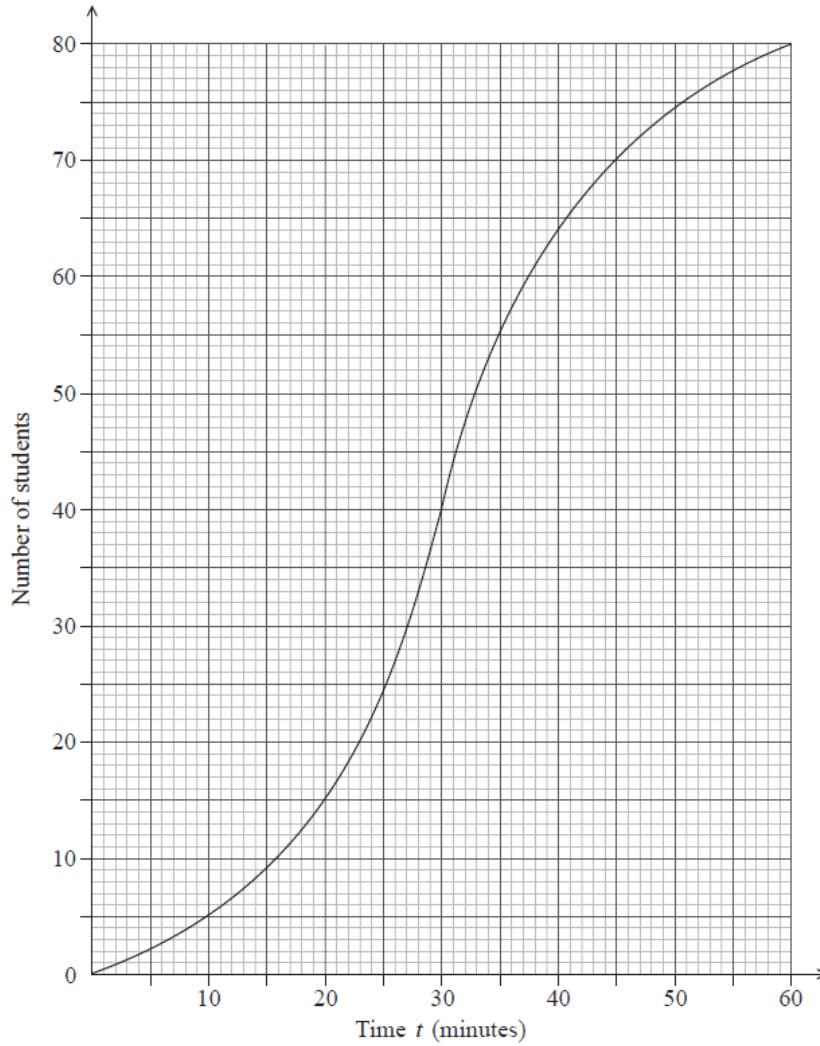
The probability that a taxicab is yellow is 0.4.

The probability that a taxicab is a Fiat is 0.3.

The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

#19

The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task.

19a. Find the number of students who completed the task in less than 45 minutes.   *[2 marks]*

19b. Find the number of students who took between 35 and 45 minutes to complete the task.   *[3 marks]*

19c. Given that 50 students take less than k minutes to complete the task, find the value of k.   *[2 marks]*

#20 (OMIT)

Let

$$A = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix} \text{ and}$$
$$B = \begin{pmatrix} -4 & 0 \\ 5 & 1 \end{pmatrix}.$$

20a. Find AB.   *[3 marks]*

20b. Given that $X - 2A = B$, find X.   *[3 marks]*

#21 The following table shows the probability distribution of a discrete random variable X.

x	0	2	5	9
$P(X = x)$	0.3	k	$2k$	0.1

21a. Find the value of k .   [3 marks]

21b. Find $E(X)$.   [3 marks]

#22 The ages of people attending a music concert are given in the table below.

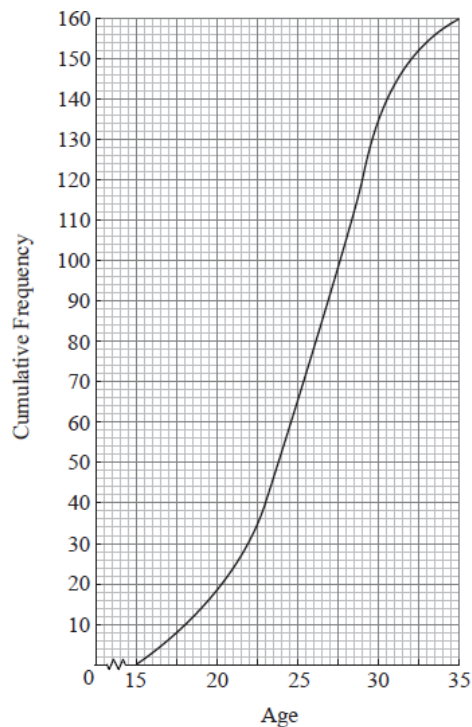| Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

22a. Find p .   [2 marks]

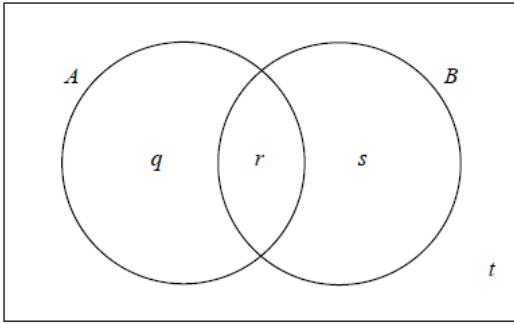22b. The cumulative frequency diagram is given below.   [5 marks]


Use the diagram to estimate

(i)   the 80th percentile;

(ii)   the interquartile range.

#23 Events *A* and *B* are such that
$P(A) = 0.3$,
$P(B) = 0.6$ and
$P(A \cup B) = 0.7$.

The values *q* , *r* , *s* and *t* represent probabilities.

23a. Write down the value of *t* .   *[1 mark]*

23b. (i)   Show that   *[3 marks]*
     $r = 0.2$.

     (ii)   Write down the value of *q* and of *s* .

23c. (i)   Write down   *[3 marks]*
     $P(B')$.

     (ii)   Find
     $P(A|B')$.

#24 The cumulative frequency curve below represents the marks obtained by 100 students.
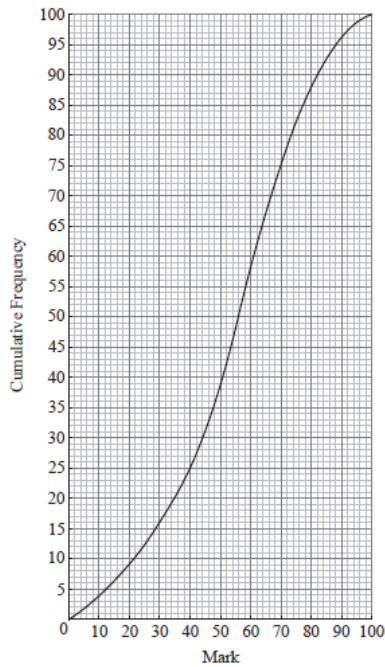
24a. Find the median mark.   *[2 marks]*

24b. Find the interquartile range.   *[3 marks]*

#25 The random variable X has the following probability distribution, with
$P(X > 1) = 0.5$.

x	0	1	2	3
$P(X = x)$	p	q	r	0.2

25a. Find the value of r.   [2 marks]

25b. Given that   [6 marks]
$E(X) = 1.4$, find the value of p and of q .

#26 A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble
and then selects a second marble.

26a. Write down the probability that the first marble Anna selects is red.   [1 mark]

26b. Find the probability that Anna selects two red marbles.   [2 marks]

26c. Find the probability that one marble is red and one marble is blue.   [3 marks]

#27 Let
$f(x) = \frac{1}{2}x^2 + kx + 8$, where
$k \in \mathbb{Z}$.

27a. Find the values of k such that   [4 marks]
$f(x) = 0$ has two equal roots.

27b. Each value of k is equally likely for   [4 marks]
$-5 \le k \le 5$. Find the probability that
$f(x) = 0$ has no roots.