

Practice quiz key

1a) $P(x) = x^4 - 12x^3 + 52x^2 - 96x + 64$

Note: $P(-x) = x^4 + 12x^3 + 52x^2 + 96x + 64 \Rightarrow$ no neg. roots

$$\begin{array}{r|rrrrr} \cancel{1} & 1 & -12 & 52 & -96 & 64 \\ & & 1 & -11 & 41 & -55 \\ \hline & 1 & -11 & 41 & -55 & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -12 & 52 & -96 & 64 \\ & & 2 & -20 & 64 & -64 \\ \hline & 1 & -10 & 32 & -32 & 0 = P(2) \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -10 & 32 & -32 \\ & & 2 & -16 & 32 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$$(x-2)^2 (x^2 - 8x + 16) = 0$$

$$(x-2)^2 (x-4)^2 = 0$$

double root double root
 ↓ ↓
 zeros: 2, 4

Descartes: 2 or 0 negs
 2 or 0 pos

1b) $P(x) = x^4 - x^2 + 2x + 2$

$P(1) = 4$

$P(-1) = 0$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -1 & 2 & 2 \\ & & -1 & 1 & 0 & -2 \\ \hline & 1 & -1 & 0 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$P(x) = (x+1)^2 (x^2 - 2x + 2) = 0$$

$$x = -1$$

$$x = 1 \pm i$$

$$x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$
$$= \frac{2 \pm 2i}{2} = \underline{1 \pm i}$$

2

$$x = 2 + 3i$$

$$x - 2 = 3i$$

$$(x-2)^2 = (3i)^2$$

$$x^2 - 4x + 4 = 9i^2 = -9$$

$$\underline{x^2 - 4x + 13 = 0}$$

$$x = -4$$

$$\underline{x + 4 = 0}$$

$$P(x) = (x+4)(x^2 - 4x + 13)$$

$$= x^3 - \cancel{4x^2} + 13x + \cancel{4x^2} - 16x + 52$$

$$P(x) = x^3 - 3x + 52$$

