

Triangle Trigonometry: The Sine Rule & the Cosine Rule

#1 $\alpha = 37^\circ$, $\beta = 91^\circ$, $a = 15$ Find b .

(AAS)

$$\frac{\sin 91^\circ}{b} = \frac{\sin 37^\circ}{15}$$

$$b \cdot \sin 37^\circ = 15 \cdot \sin 91^\circ$$

$$b = \frac{15 \cdot \sin 91^\circ}{\sin 37^\circ} \approx \underline{24.9}$$

#2 $a = 9$, $b = 40$, $c = 39$ Find α .

(SSS)

opposites

$$\cos \alpha = \frac{40^2 + 39^2 - 9^2}{2(40)(39)} \approx 0.9744$$

$$\alpha = \cos^{-1}(0.9744) = \underline{13.0^\circ}$$

#3 $\alpha = 28^\circ$, $\gamma = 63^\circ$, $b = 12$ Find c .

(ASA) $\beta = 180^\circ - 63^\circ - 28^\circ = 89^\circ$ (opposite from b)

$$\frac{\sin 63^\circ}{c} = \frac{\sin 89^\circ}{12}$$

$$c \cdot \sin 89^\circ = 12 \sin 63^\circ$$

$$c = \frac{12 \sin 63^\circ}{\sin 89^\circ} = \underline{\underline{10.7}}$$

#4 $\gamma = 79^\circ$, $a = 11$, $c = 9$. Find α

(SSA - so, watch out!)

$$\frac{\sin \alpha}{11} = \frac{\sin 79^\circ}{9}$$

$$\sin \alpha = \frac{11 \cdot \sin 79^\circ}{9} \approx 1.2 \rightarrow \boxed{\text{NO SUCH TRIANGLE}}$$

#5 $\beta = 99^\circ$, $\gamma = 58^\circ$, $b = 20$ Find c

(AAS) $\frac{\sin 58^\circ}{c} = \frac{\sin 99^\circ}{20}$

$$c = \frac{20 \cdot \sin 58^\circ}{\sin 99^\circ} \approx \underline{\underline{17.2}}$$

#6 $b=21$, $\alpha=25^\circ$, $c=13$, Find a .
(SAS)

$$a^2 = 21^2 + 13^2 - 2(21)(13) \cos 25^\circ$$

↑ ↑
opposites

$$a^2 = 115.156 \Rightarrow \underline{\underline{a = 10.7}}$$

#7 $a=6$, $b=15$, $c=18$ Find β

(SSS)

$$\cos \beta = \frac{6^2 + 18^2 - 15^2}{2(6)(18)} \approx 0.625$$

$$\underline{\underline{\beta = 51.3^\circ}}$$

#8 $\beta=89^\circ$, $\gamma=57^\circ$, $a=3$, Find b

(ASA) $\alpha = 180^\circ - 89^\circ - 57^\circ = 34^\circ$

$$\frac{\sin 89^\circ}{b} = \frac{\sin 34^\circ}{3}$$

$$b = \frac{3 \cdot \sin 89^\circ}{\sin 34^\circ} \approx \underline{\underline{5.4}}$$

#9 $\gamma = 39^\circ$, $a = 11$, $c = 13$. Find α .
(SSA - be careful!)

$$\frac{\sin \alpha}{11} = \frac{\sin 39^\circ}{13}$$

$$\sin \alpha = \frac{11 \cdot \sin 39^\circ}{13} = 0.8460$$

$$\alpha = 57.8^\circ \text{ or } 180 - 57.8^\circ = 122.2^\circ$$

$+ 39$

 161.2

$\alpha = 57.8^\circ \text{ or } 122.2^\circ$

#10 $\gamma = 80^\circ$, $a = 10$, $c = 4$. Find α .
(SSA - watch your step!)

$$\frac{\sin \alpha}{10} = \frac{\sin 80^\circ}{4}$$

$$\sin \alpha = \frac{10 \cdot \sin 80^\circ}{4} \approx 2.5 \rightarrow \boxed{\text{NO SUCH TRIANGLE}}$$

#11 $\alpha = 24^\circ$, $\beta = 115^\circ$, $c = 44$ Find b

(ASA) $\gamma = 180^\circ - 24^\circ - 115^\circ = 41^\circ$

$$\frac{\sin 115^\circ}{b} = \frac{\sin 41^\circ}{44}$$

$$b = \frac{44 \cdot \sin 115^\circ}{\sin 41^\circ} \approx \underline{\underline{60.8}}$$