5.1 In Exercises 1–6, name the trigonometric function that is equivalent to the expression.

1. $\dfrac{\sin x}{\cos x}$

2. $\dfrac{1}{\sin x}$

3. $\dfrac{1}{\sec x}$

4. $\dfrac{1}{\tan x}$

5. $\sqrt{\cot^2 x + 1}$

6. $\sqrt{1 + \tan^2 x}$

In Exercises 7–10, use the given values and trigonometric identities to evaluate (if possible) all six trigonometric functions.

7. $\sin x = \frac{5}{13}, \quad \cos x = \frac{12}{13}$

8. $\tan \theta = \dfrac{2}{3}, \quad \sec \theta = \dfrac{\sqrt{13}}{3}$

9. $\sin\left(\dfrac{\pi}{2} - x\right) = \dfrac{\sqrt{2}}{2}, \quad \sin x = -\dfrac{\sqrt{2}}{2}$

10. $\csc\left(\dfrac{\pi}{2} - \theta\right) = 9, \quad \sin \theta = \dfrac{4\sqrt{5}}{9}$

In Exercises 11–24, use the fundamental trigonometric identities to simplify the expression.

11. $\dfrac{1}{\cot^2 x + 1}$

12. $\dfrac{\tan \theta}{1 - \cos^2 \theta}$

13. $\tan^2 x(\csc^2 x - 1)$

14. $\cot^2 x(\sin^2 x)$

15. $\dfrac{\sin\left(\dfrac{\pi}{2} - \theta\right)}{\sin \theta}$

16. $\dfrac{\cot\left(\dfrac{\pi}{2} - u\right)}{\cos u}$

17. $\dfrac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$

18. $\dfrac{\sec^2(-\theta)}{\csc^2 \theta}$

19. $\cos^2 x + \cos^2 x \cot^2 x$

20. $\tan^2 \theta \csc^2 \theta - \tan^2 \theta$

21. $(\tan x + 1)^2 \cos x$

22. $(\sec x - \tan x)^2$

23. $\dfrac{1}{\csc \theta + 1} - \dfrac{1}{\csc \theta - 1}$

24. $\dfrac{\tan^2 x}{1 + \sec x}$

In Exercises 25 and 26, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

25. $\sqrt{25 - x^2}, \ x = 5 \sin \theta$

26. $\sqrt{x^2 - 16}, \ x = 4 \sec \theta$

27. **RATE OF CHANGE** The rate of change of the function $f(x) = \csc x - \cot x$ is given by the expression $\csc^2 x - \csc x \cot x$. Show that this expression can also be written as

$$\frac{1 - \cos x}{\sin^2 x}.$$

28. **RATE OF CHANGE** The rate of change of the function $f(x) = 2\sqrt{\sin x}$ is given by the expression $\sin^{-1/2} x \cos x$. Show that this expression can also be written as $\cot x \sqrt{\sin x}$.

5.2 In Exercises 29–36, verify the identity.

29. $\cos x(\tan^2 x + 1) = \sec x$

30. $\sec^2 x \cot x - \cot x = \tan x$

31. $\sec\left(\dfrac{\pi}{2} - \theta\right) = \csc \theta$

32. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

33. $\dfrac{1}{\tan \theta \csc \theta} = \cos \theta$

34. $\dfrac{1}{\tan x \csc x \sin x} = \cot x$

35. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x$

36. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x) \cos x$

5.3 In Exercises 37–42, solve the equation.

37. $\sin x = \sqrt{3} - \sin x$

38. $4 \cos \theta = 1 + 2 \cos \theta$

39. $3\sqrt{3} \tan u = 3$

40. $\frac{1}{2} \sec x - 1 = 0$

41. $3 \csc^2 x = 4$

42. $4 \tan^2 u - 1 = \tan^2 u$

In Exercises 43–52, find all solutions of the equation in the interval $[0, 2\pi)$.

43. $2\cos^2 x - \cos x = 1$

44. $2\sin^2 x - 3 \sin x = -1$

45. $\cos^2 x + \sin x = 1$

46. $\sin^2 x + 2 \cos x = 2$

47. $2 \sin 2x - \sqrt{2} = 0$

48. $2 \cos \dfrac{x}{2} + 1 = 0$

49. $3 \tan^2\left(\dfrac{x}{3}\right) - 1 = 0$

50. $\sqrt{3} \tan 3x = 0$

51. $\cos 4x(\cos x - 1) = 0$

52. $3 \csc^2 5x = -4$

In Exercises 53–56, use inverse functions where needed to find all solutions of the equation in the interval $[0, 2\pi)$.

53. $\sin^2 x - 2 \sin x = 0$

54. $2\cos^2 x + 3 \cos x = 0$

55. $\tan^2 \theta + \tan \theta - 6 = 0$

56. $\sec^2 x + 6 \tan x + 4 = 0$

5.4 In Exercises 57–60, find the exact values of the sine, cosine, and tangent of the angle.

57. $285° = 315° - 30°$

58. $345° = 300° + 45°$

59. $\dfrac{25\pi}{12} = \dfrac{11\pi}{6} + \dfrac{\pi}{4}$

60. $\dfrac{19\pi}{12} = \dfrac{11\pi}{6} - \dfrac{\pi}{4}$

In Exercises 61–64, write the expression as the sine, cosine, or tangent of an angle.

61. $\sin 60° \cos 45° - \cos 60° \sin 45°$

62. $\cos 45° \cos 120° - \sin 45° \sin 120°$

63. $\dfrac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°}$

64. $\dfrac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

In Exercises 65–70, find the exact value of the trigonometric function given that $\tan u = \frac{3}{4}$ and $\cos v = -\frac{4}{5}$. (u is in Quadrant I and v is in Quadrant III.)

65. $\sin(u + v)$ **66.** $\tan(u + v)$

67. $\cos(u - v)$ **68.** $\sin(u - v)$

69. $\cos(u + v)$ **70.** $\tan(u - v)$

In Exercises 71–76, verify the identity.

71. $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$ **72.** $\sin\left(x - \dfrac{3\pi}{2}\right) = \cos x$

73. $\tan\left(x - \dfrac{\pi}{2}\right) = -\cot x$ **74.** $\tan(\pi - x) = -\tan x$

75. $\cos 3x = 4 \cos^3 x - 3 \cos x$

76. $\dfrac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} = \dfrac{\tan \alpha - \tan \beta}{\tan \alpha + \tan \beta}$

In Exercises 77–80, find all solutions of the equation in the interval $[0, 2\pi)$.

77. $\sin\left(x + \dfrac{\pi}{4}\right) - \sin\left(x - \dfrac{\pi}{4}\right) = 1$

78. $\cos\left(x + \dfrac{\pi}{6}\right) - \cos\left(x - \dfrac{\pi}{6}\right) = 1$

79. $\sin\left(x + \dfrac{\pi}{2}\right) - \sin\left(x - \dfrac{\pi}{2}\right) = \sqrt{3}$

80. $\cos\left(x + \dfrac{3\pi}{4}\right) - \cos\left(x - \dfrac{3\pi}{4}\right) = 0$

5.5 In Exercises 81–84, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

81. $\sin u = -\dfrac{4}{5}, \quad \pi < u < \dfrac{3\pi}{2}$

82. $\cos u = -\dfrac{2}{\sqrt{5}}, \quad \dfrac{\pi}{2} < u < \pi$

83. $\sec u = -3, \quad \dfrac{\pi}{2} < u < \pi$

84. $\cot u = 2, \quad \pi < u < \dfrac{3\pi}{2}$

In Exercises 85 and 86, use double-angle formulas to verify the identity algebraically and use a graphing utility to confirm your result graphically.

85. $\sin 4x = 8 \cos^3 x \sin x - 4 \cos x \sin x$

86. $\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

In Exercises 87–90, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

87. $\tan^2 2x$ **88.** $\cos^2 3x$

89. $\sin^2 x \tan^2 x$ **90.** $\cos^2 x \tan^2 x$

In Exercises 91–94, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

91. $-75°$ **92.** $15°$

93. $\dfrac{19\pi}{12}$ **94.** $-\dfrac{17\pi}{12}$

In Exercises 95–98, (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

95. $\sin u = \frac{7}{25}, \quad 0 < u < \pi/2$

96. $\tan u = \frac{4}{3}, \quad \pi < u < 3\pi/2$

97. $\cos u = -\frac{2}{7}, \quad \pi/2 < u < \pi$

98. $\sec u = -6, \quad \pi/2 < u < \pi$

In Exercises 99 and 100, use the half-angle formulas to simplify the expression.

99. $-\sqrt{\dfrac{1 + \cos 10x}{2}}$ **100.** $\dfrac{\sin 6x}{1 + \cos 6x}$

In Exercises 101–104, use the product-to-sum formulas to write the product as a sum or difference.

101. $\cos \dfrac{\pi}{6} \sin \dfrac{\pi}{6}$ **102.** $6 \sin 15° \sin 45°$

103. $\cos 4\theta \sin 6\theta$ **104.** $2 \sin 7\theta \cos 3\theta$

In Exercises 105–108, use the sum-to-product formulas to write the sum or difference as a product.

105. $\sin 4\theta - \sin 8\theta$

106. $\cos 6\theta + \cos 5\theta$

107. $\cos\left(x + \dfrac{\pi}{6}\right) - \cos\left(x - \dfrac{\pi}{6}\right)$

108. $\sin\left(x + \dfrac{\pi}{4}\right) - \sin\left(x - \dfrac{\pi}{4}\right)$