Quiz on continuous distributions name:

CALCULATOR

[1] The probability density function for a random variable X is defined by $f(x)=\begin{cases}\frac{4x^3}{c}, & 1\le x\le 4\\ 0, & \text{elsewhere}\end{cases}$.

[a] Find the value of c. $\int_1^4 \frac{4x^2}{c}dx=1$ **use the solver to get c = 255**

[b] Find E(X). $E(X)=\int_1^4 \frac{4x^4}{255}dx=\frac{1364}{425}$

[c] Find Var(X) $Var(X)=\int_1^4\left(x-\frac{1364}{425}\right)^2\cdot\frac{4x^3}{255}dx=0.406$

[d] Find the median value of X. $\int_1^m \frac{4x^2}{255}dx=\frac{1}{2}$ **use the solver to get *m* = 3.37**

[e] Find $P(2<X<3)$ $\int_2^3 \frac{4x^2}{255}dx=\frac{13}{51}$

NO CALCULATOR

[2] The probability density function for a random variable Y is defined by $g(x)=\begin{cases}3x^2, & 0\le x\le 1\\ 0, & \text{elsewhere}\end{cases}$.

[a] Find the variance of Y.

[b] Find $P\left(0<X<\frac{1}{2}\right)$

(a) $E(Y)=\int_0^1 x\cdot 3x^2dx=\int_0^1 3x^3dx=\left[\frac{3}{4}x^4\right]_0^1=\frac{3}{4}$

$E(Y^2)=\int_0^1 x^2\cdot 3x^2dx=\int_0^1 3x^4dx=\left[\frac{3}{5}x^5\right]_0^1=\frac{3}{5}$

$Var(Y)=\frac{3}{5}-\left(\frac{3}{4}\right)^2=\frac{3}{5}-\frac{9}{16}=\frac{48}{80}-\frac{45}{80}=\frac{3}{80}$

(b) $P(0<x<\frac{1}{2})=\int_0^{\frac{1}{2}}3x^2dx=\left[x^3\right]_0^{1/2}=\left(\frac{1}{2}\right)^3-0^3=\frac{1}{8}$