Optional Practice

1. (a) $f(x) = 1 - 2x - 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{[1 - 2(x+h) - 3(x+h)^2] - [1 - 2x - 3x^2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} - \cancel{2x} - 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{1} + \cancel{2x} + \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[-2 - 6x - 3h]}{\cancel{h}} = -2 - 6x$$

(b) $f(x) = \dfrac{2x+1}{x-1}$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{2(x+h)+1}{(x+h)-1} - \dfrac{2x+1}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(2x+2h+1)(x-1)}{(x+h-1)(x-1)} - \dfrac{(2x+1)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} - \cancel{2x} + \cancel{2xh} - 2h + \cancel{x} - \cancel{1} - (\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{x} + h - \cancel{1})}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(x+h-1)(x-1)} = \frac{-3}{(x-1)^2}$$

(c) $f(x) = \sqrt{5-2x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{5-2(x+h)} - \sqrt{5-2x}}{h} \cdot \frac{\sqrt{5-2(x+h)} + \sqrt{5-2x}}{\sqrt{5-2(x+h)} + \sqrt{5-2x}}$$

$$= \lim_{h \to 0} \frac{5-2(x+h) - (5-2x)}{h\left(\sqrt{5-2(x+h)} + \sqrt{5-2x}\right)} = \lim_{h \to 0} \frac{\cancel{5} - \cancel{2x} - 2h - \cancel{5} + \cancel{2x}}{h\left(\sqrt{5-2(x+h)} + \sqrt{5-2x}\right)}$$

$$= \lim_{h \to 0} \frac{-2}{\sqrt{5-2(x+h)} + \sqrt{5-2x}} = \frac{-2}{2\sqrt{5-2x}} = \frac{-1}{\sqrt{5-2x}}$$

2(a) $f(x) = 2x^3 + x^2$ at $x = 2$

$$f'(2) = \lim_{x \to 2} \frac{(2x^3 + x^2) - (2 \cdot 2^3 + 2^2)}{x-2}$$

$$= \lim_{x \to 2} \frac{2x^3 + x^2 - 20}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & 0 & -20 \\ & & 4 & 10 & 20 \\ \hline & 2 & 5 & 10 & \boxed{0} \end{array}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(2x^2 + 5x + 10)}{\cancel{x-2}} = 2(2)^2 + 5(2) + 10$$

$$= \underline{\underline{28}}$$