

[2] Consider the polynomial  $f(x) = 2x^4 - 3x^3 + 7x^2 + 7x - 5$ . The roots of this polynomial are  $x_1, x_2, x_3, x_4$  and  $x_3$ .

[a] Without solving, find the value of  $x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$

$$\frac{7}{2}$$

+ - + + - 3 or 1 pos  
 + + + - - 1 neg root

[b] Factor the polynomial and state its roots.

$$\begin{array}{r} 1 \mid 2 \quad -3 \quad 7 \quad 7 \quad -5 \\ \quad \quad 2 \quad -1 \quad 6 \quad 13 \\ \hline 2 \quad -1 \quad 6 \quad 13 \end{array}$$

$$\begin{array}{r} -1 \mid 2 \quad -3 \quad 7 \quad 7 \quad -5 \\ \quad \quad -2 \quad 5 \quad -12 \quad 5 \\ \hline 2 \quad -5 \quad 12 \quad -5 \quad 0 \end{array} \quad \text{no more negatives}$$

$$\begin{array}{r} 5 \mid 2 \quad -5 \quad 12 \quad -5 \\ \quad \quad 10 \quad 25 \\ \hline 2 \quad 5 \quad 37 \quad \text{pos} \\ \text{no larger roots} \end{array}$$

$$\begin{array}{r} \frac{1}{2} \mid 2 \quad -5 \quad 12 \quad -5 \\ \quad \quad 1 \quad -2 \quad 5 \\ \hline 2 \quad -4 \quad 10 \quad 0 \end{array}$$

$$(x+1)(2x-1)(x^2-2x+5) = 0$$

$$x = -1 \quad x = \frac{1}{2}$$

$$x = 1 \pm 2i$$

$$x = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm 4i}{2}$$

Quiz - Roots of polynomials

name:

[1] Consider the polynomial  $f(x) = 3x^3 - 7x^2 + 8x - 2$ . The roots of this polynomial are  $x_1, x_2,$  and  $x_3$ .

[a] Without solving, find the value of  $x_1^2 + x_2^2 + x_3^2$

$$(x_1 + x_2 + x_3)^2 = (x_1^2 + x_2^2 + x_3^2) + 2(x_1x_2 + x_1x_3 + x_2x_3)$$

$$\left(\frac{7}{3}\right)^2 = (x_1^2 + x_2^2 + x_3^2) + 2\left(\frac{8}{3}\right)$$

$$\frac{49}{9} - \frac{48}{9} = \frac{1}{9} = x_1^2 + x_2^2 + x_3^2$$

+ - + -  
- - - - no negative roots

[b] List the possible linear factors:  $x-1, x-2, 3x-1, 3x-2$

[c] Factor the polynomial and state its roots.

$$\begin{array}{r} \underline{1} \quad 3 \quad -7 \quad 8 \quad -2 \\ \quad \quad 3 \quad -4 \quad 4 \\ \hline 3 \quad -4 \quad 4 \end{array}$$

$$\begin{array}{r} \underline{2} \quad 3 \quad -7 \quad 8 \quad -2 \\ \quad \quad 6 \quad -2 \quad 12 \\ \hline 3 \quad -1 \quad 6 \quad 10 \end{array}$$

$$\begin{array}{r} \underline{\frac{1}{3}} \quad 3 \quad -7 \quad 8 \quad -2 \\ \quad \quad 1 \quad -2 \quad 2 \\ \hline 3 \quad -6 \quad 6 \quad 0 \end{array}$$

$$(3x-1)(x^2-2x+2) = 0$$

$$x = -\frac{1}{3} \quad x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$$x = 1 \pm i$$