

Calculus

Differentiate each function with respect to the given variable.

$$1) h = (t^4 - 3)\sqrt[5]{t + 3}$$

$$\begin{aligned}\frac{dh}{dt} &= (t^4 - 3) \cdot \frac{1}{5}(t + 3)^{-\frac{4}{5}} + (t + 3)^{\frac{1}{5}} \cdot 4t^3 \\ &= \frac{3(7t^4 + 20t^3 - 1)}{5(t + 3)^{\frac{4}{5}}}\end{aligned}$$

$$2) t = \sqrt[3]{\frac{2x^3 + 5}{-3x^5 + 2}}$$

$$\begin{aligned}\frac{dt}{dx} &= \frac{1}{3} \cdot \left(\frac{2x^3 + 5}{-3x^5 + 2}\right)^{-\frac{2}{3}} \cdot \frac{(-3x^5 + 2) \cdot 6x^2 - (2x^3 + 5) \cdot (-15x^4)}{(-3x^5 + 2)^2} \\ &= \frac{x^2(4x^5 + 4 + 25x^2)}{(2x^3 + 5)^{\frac{2}{3}} \cdot (-3x^5 + 2)^{\frac{4}{3}}}\end{aligned}$$

$$3) f = \frac{\sqrt[5]{x^3 - 4}}{3x^5 + 2}$$

$$\begin{aligned}\frac{df}{dx} &= \frac{(3x^5 + 2) \cdot \frac{1}{5}(x^3 - 4)^{-\frac{4}{5}} \cdot 3x^2 - (x^3 - 4)^{\frac{1}{5}} \cdot 15x^4}{(3x^5 + 2)^2} \\ &= \frac{6x^2(-11x^5 + 50x^2 + 1)}{5(3x^5 + 2)^2 \cdot (x^3 - 4)^{\frac{4}{5}}}\end{aligned}$$

$$4) y = \frac{(-5t^5 - 3)^{\frac{1}{3}}}{\sqrt[5]{5t^2 + 1}}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{(5t^2 + 1)^{\frac{1}{5}} \cdot \frac{1}{3}(-5t^5 - 3)^{-\frac{2}{3}} \cdot -25t^4 - (-5t^5 - 3)^{\frac{1}{3}} \cdot \frac{1}{5}(5t^2 + 1)^{-\frac{4}{5}} \cdot 10t}{\left((5t^2 + 1)^{\frac{1}{5}}\right)^2} \\ &= \frac{t(-95t^5 - 25t^3 + 18)}{3(-5t^5 - 3)^{\frac{2}{3}} \cdot (5t^2 + 1)^{\frac{6}{5}}} \end{aligned}$$

$$5) h = \frac{(4t^4 + 5)^{\frac{1}{3}}}{3t - 2}$$

$$\begin{aligned} \frac{dh}{dt} &= \frac{(3t - 2) \cdot \frac{1}{3}(4t^4 + 5)^{-\frac{2}{3}} \cdot 16t^3 - (4t^4 + 5)^{\frac{1}{3}} \cdot 3}{(3t - 2)^2} \\ &= \frac{12t^4 - 45 - 32t^3}{3(3t - 2)^2 \cdot (4t^4 + 5)^{\frac{2}{3}}} \end{aligned}$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

$$6) 2 = x^2 + 3x^2y^3 + x^3y$$

$$\frac{dy}{dx} = \frac{-2 - 6y^3 - 3xy}{9xy^2 + x^2}$$

$$7) -xy^3 + 2 = 2x^2 + xy$$

$$\frac{dy}{dx} = \frac{4x + y + y^3}{-3xy^2 - x}$$

$$8) 1 = 2x^3 + 3x^3y + xy$$

$$\frac{dy}{dx} = \frac{6x^2 + 9x^2y + y}{-3x^3 - x}$$

$$9) -4y^2 - 5x^3y^2 + 3 = 3x^2$$

$$\frac{dy}{dx} = \frac{6x + 15x^2y^2}{-8y - 10yx^3}$$

$$10) 2x^3 + 4y + 4x^3y^2 = 4$$

$$\frac{dy}{dx} = \frac{-3x^2 - 6x^2y^2}{2 + 4x^3y}$$

Differentiate each function with respect to x .

11) $y = \sin^{-1}(x^5 - 5)^4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sqrt{1 - ((x^5 - 5)^4)^2}} \cdot 4(x^5 - 5)^3 \cdot 5x^4 \\ &= \frac{20x^4(x^5 - 5)^3}{\sqrt{1 - (x^5 - 5)^8}}\end{aligned}$$

12) $y = (\tan^{-1} 3x^3)^2$

$$\begin{aligned}\frac{dy}{dx} &= 2\tan^{-1} 3x^3 \cdot \frac{1}{(3x^3)^2 + 1} \cdot 9x^2 \\ &= \frac{18x^2 \tan^{-1} 3x^3}{9x^6 + 1}\end{aligned}$$

13) $y = (\tan^{-1} 5x^4)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4(\tan^{-1} 5x^4)^3 \cdot \frac{1}{(5x^4)^2 + 1} \cdot 20x^3 \\ &= \frac{80x^3 (\tan^{-1} 5x^4)^3}{25x^8 + 1}\end{aligned}$$

14) $y = \frac{\ln 4x^3}{\sqrt[3]{4x^5 - 5}}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(4x^5 - 5)^{\frac{1}{3}} \cdot \frac{1}{4x^3} \cdot 12x^2 - \ln 4x^3 \cdot \frac{1}{3}(4x^5 - 5)^{-\frac{2}{3}} \cdot 20x^4}{\left((4x^5 - 5)^{\frac{1}{3}}\right)^2} \\ &= \frac{36x^5 - 45 - 20x^5 \ln 4x^3}{3x \sqrt[3]{(4x^5 - 5)^4}}\end{aligned}$$

$$15) y = \frac{e^{3x^5}}{\sin x^4}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sin x^4 \cdot e^{3x^5} \cdot 15x^4 - e^{3x^5} \cdot \cos x^4 \cdot 4x^3}{\sin^2 x^4} \\ &= \frac{x^3 e^{3x^5} (15x \sin x^4 - 4 \cos x^4)}{\sin^2 x^4} \end{aligned}$$

$$16) y = e^{3x^4} \cdot (5x^5 - 3)^4$$

$$\begin{aligned} \frac{dy}{dx} &= e^{3x^4} \cdot 4(5x^5 - 3)^3 \cdot 25x^4 + (5x^5 - 3)^4 \cdot e^{3x^4} \cdot 12x^3 \\ &= 4x^3 e^{3x^4} \cdot (5x^5 - 3)^3 (25x + 15x^5 - 9) \end{aligned}$$

$$17) y = \ln 4x^2 \cdot \sec 4x^4$$

$$\begin{aligned} \frac{dy}{dx} &= \ln 4x^2 \cdot \sec 4x^4 \tan 4x^4 \cdot 16x^3 + \sec 4x^4 \cdot \frac{1}{4x^2} \cdot 8x \\ &= \frac{2 \sec 4x^4 \cdot (8x^4 \tan 4x^4 \cdot \ln 4x^2 + 1)}{x} \end{aligned}$$

$$18) y = \frac{e^{4x^5}}{\sqrt[5]{5x^3 + 2}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(5x^3 + 2)^{\frac{1}{5}} \cdot e^{4x^5} \cdot 20x^4 - e^{4x^5} \cdot \frac{1}{5} (5x^3 + 2)^{-\frac{4}{5}} \cdot 15x^2}{\left((5x^3 + 2)^{\frac{1}{5}} \right)^2} \\ &= \frac{x^2 e^{4x^5} (100x^5 + 40x^2 - 3)}{\sqrt[5]{(5x^3 + 2)^6}} \end{aligned}$$