

## Review Exercises -- Logarithms

**[1]** Simplify each expression without a calculator.

[a]  $9^{-1}$

[b]  $9^{-2}$

[c]  $9^0$

[d]  $9^3$

[e]  $9^{\frac{1}{2}}$

[f]  $9^{-\frac{1}{2}}$

[g]  $9^{\frac{3}{2}}$

[h]  $9^{-\frac{3}{2}}$

[i]  $\left(\frac{8}{125}\right)^{\frac{1}{3}}$

[j]  $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$

[k]  $\left(\frac{8}{125}\right)^{\frac{2}{3}}$

[l]  $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

**[2]** Rewrite each exponential expression as a logarithm.

[a]  $4^3 = 64$

[b]  $2^5 = 32$

[c]  $e^{\frac{1}{2}} = \sqrt{e}$

[d]  $e^0 = 1$

**[3]** Rewrite each logarithm as an exponential expression.

[a]  $\log_3 27 = 3$

[b]  $\log_{10} \sqrt{10} = \frac{1}{2}$

[c]  $\ln e^2 = 2$

[d]  $\ln \frac{1}{e} = -1$

**[4]** Evaluate each logarithm without a calculator.

[a]  $\log_2 4 =$

[b]  $\log_4 2 =$

[c]  $\log_5 1 =$

[d]  $\log_2 \frac{1}{2} =$

[e]  $\log_{10} \sqrt{10} =$

[f]  $\log_6 \frac{1}{\sqrt{6}} =$

[g]  $\log_8 8 =$

[h]  $\log_8 4 =$

[i]  $\log_3 81 =$

[j]  $\log_3 \frac{1}{3} =$

[k]  $\log_3 3 =$

[l]  $\log_3 1 =$

[m]  $\log_3 \sqrt[3]{3} =$

[n]  $\log_3 \frac{1}{\sqrt{3}} =$

[o]  $\log_3 \frac{1}{9} =$

[p]  $\log_3 3^5 =$

[q]  $\ln e =$

[r]  $\ln \frac{1}{e} =$

[s]  $\ln e^3 =$

[t]  $\ln \sqrt{e} =$

**[5]** Rewrite each expression as a single logarithm.

[a]  $\log_2 3 + \log_2 y =$

[b]  $2\log_2 5 + 3\log_2 x =$

[c]  $\log_3 p - \log_3 q =$

[d]  $3\log_3 x - 4\log_3 y =$

[e]  $\ln x - \ln 5 =$

[f]  $2\ln 4 - 3\ln y =$

**[6]** Expand each expression to as many logarithms as possible. Do not leave exponents in final answers.

[a]  $\log_2 4x$

[b]  $\log_2 \frac{p}{q}$

[c]  $\log_3 4x^3$

[d]  $\log_3 \frac{x^4}{y^6}$

[e]  $\ln \frac{x}{5yz}$

[f]  $\ln \frac{wx^2}{y^3z^4}$

**[7]** Solve each equation.

[a]  $\log_2 5 + \log_2 x = 3$

[b]  $\log_3 x + \log_3 (x-2) = 1$

[c]  $\ln x + \ln 3 = 1$

[d]  $2^{5x-1} = 4$

[e]  $5^{x^2} = 25$

[f]  $9^x = 27^{5x+2}$

[g]  $5^{2x} = 6^{x-3}$

[h]  $e^{2x+3} = 10$

[i]  $2 \times 3^{2x+1} = 5 \times 3^{x-1}$

[h]  $1 + \log_4 (x+1) = \log_4 (5x-11)$

[i]  $e^{2x} - 3e^x - 10 = 0$

[j]  $2e^{2x} - 7e^x + 3 = 0$

**[8]** Radon-222 has a half-life of 3.8235 days. How long will it take for an initial sample to decay to 10% of its initial value.

**[9]** After 10 days, a 100g sample of phosphorus-32 has decayed to 61.57g. [a] Find the half-life of phosphorus-32. [b] How long will it take the 100g sample to decay to only 1.00g? [c] Find the decay rate.

**[10]** A small town has a population of 12342 people and its population is growing at 2.13% per year. [a] Project the population 4 years from now. [b] How long will it take the population to reach 15000?

**[11]** A large city had a population of 542,900 five years ago. Due to economic decline, its current population is 524,700. [a] Find the rate of decline for this population. [b] Predict the population in 9 years. [c] How long will it take the population to decline to half a million?