

Rational Functions -- Advanced

For each function, find [a] the x-intercept(s), [b] y-intercept, [c] vertical asymptote(s), and [d] non-vertical asymptote. Make a sketch of the function.

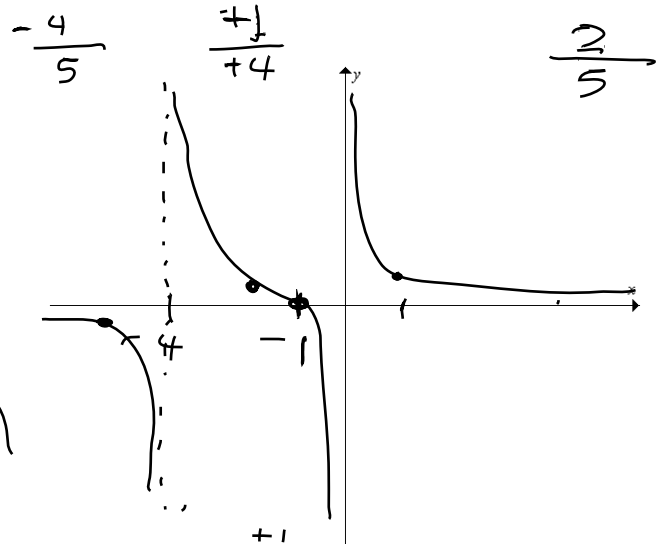
[1] $f(x) = \frac{x+1}{x^2+4x} = \frac{x+1}{x(x+4)}$

[a] -1

[b] none

[c] $x=0, x=-4$

[d] $y=0$ (bigger power in the denominator)



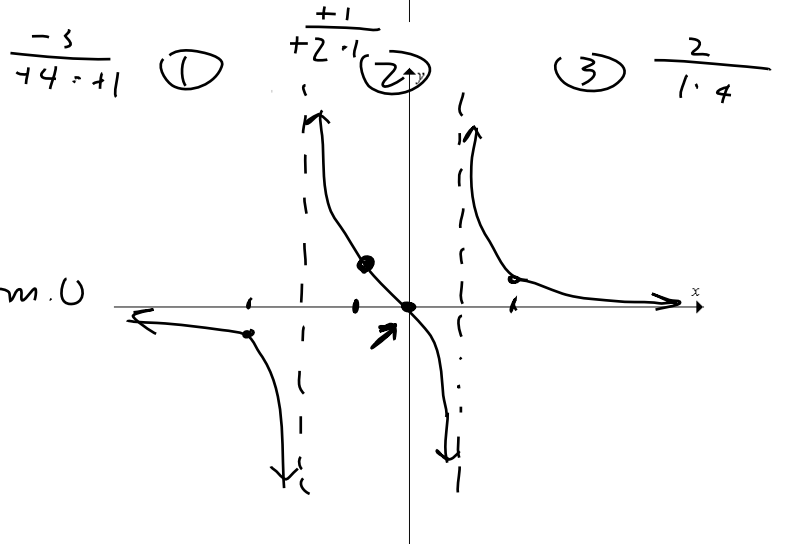
[2] $f(x) = \frac{x}{x^2+x-2} = \frac{x}{(x-1)(x+2)}$

[a] 0

[b] 0

[c] $x=1, x=-2$ (make the denom. 0)

[d] $y=0$ ($x=2$ is)



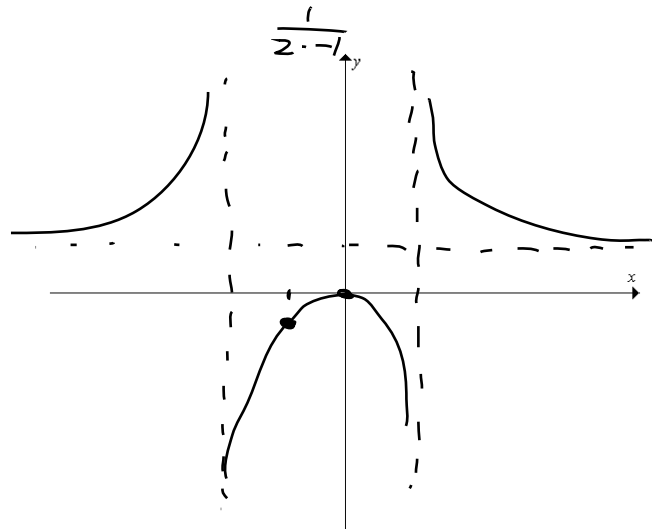
[3] $f(x) = \frac{1x^2}{x^2+x-6} = \frac{x^2}{(x+3)(x-2)}$

[a] 0

[b] 0

[c] $x=2, x=-3$

[d] $y=1$



$$[4] f(x) = \frac{x^2 + x - 12}{x^2 - 4} = \frac{(x+4)(x-3)}{(x-2)(x+2)}$$

[a] 3, -4

[b] 3

[c] $x = \pm 2$

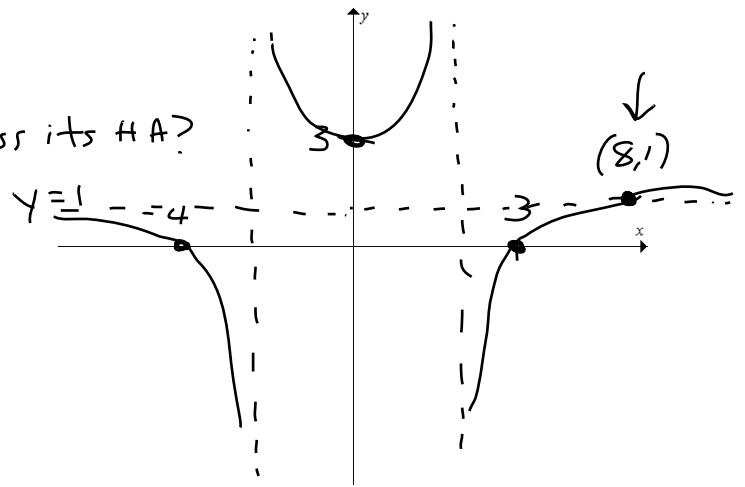
[d] $y = 1$

does the graph cross its HA?

$$\frac{x^2 + x - 12}{x^2 - 4} = 1$$

$$x^2 + x - 12 = x^2 - 4$$

$$x = 8$$



$$[5] f(x) = \frac{x^3 - 1}{x^2 - 9} = \frac{(x-1)(x^2 + x + 1)}{(x-3)(x+3)}$$

[a] 1

[b] 1/9

[c] $x = \pm 3$

[d] $y = x$

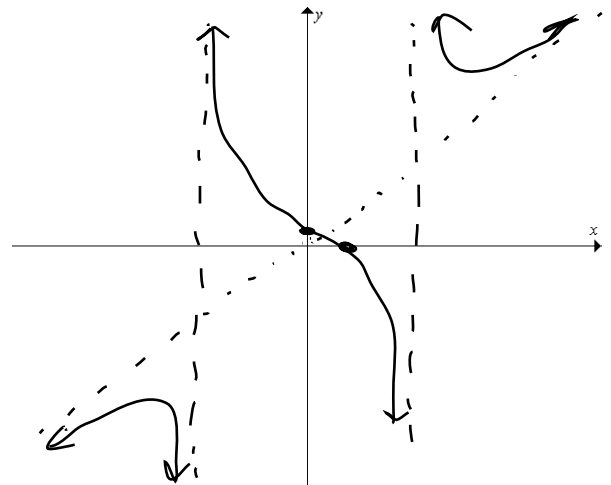
bigger power on top

$$x^2 - 9 \overline{) x^3 - 1}$$

$$\underline{-x^3 + 9x}$$

$$9x - 1$$

degree 1



$$[6] f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^3 - 8x^2 + 16x} = \frac{(x-1)(x-3)(x+2)}{x(x-4)^2}$$

[a] 1, 3, -2

[b] none

[c] $x = 0, x = 4$

[d] $y = 1$

$$\begin{array}{r} \downarrow \quad -2 \quad -5 \quad 6 \\ \quad 1 \quad -1 \quad -6 \\ \hline \quad 1 \quad -1 \quad -6 \quad 0 \end{array}$$

$(x-1)(x^2 - x - 6)$
 $(x-1)(x-3)(x+2)$

$$x^3 - 8x^2 + 16x$$

$$x(x^2 - 8x + 16)$$

$$x(x-4)(x-4)$$

