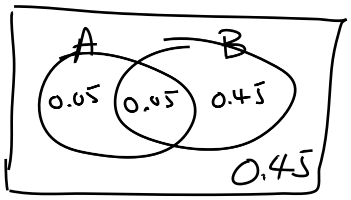
Test -- Probability name ________________________________

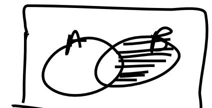
[1] For events A and B it is known that $P(A)=0.1$, $P(B)=0.5$, and $\underline{P(A'\cap B')=0.45}$.

$P(A\cup B) = 1-0.45 = 0.55$



$P(A\cap B) = P(A)+P(B)-P(A\cup B)$ ✱

$= 0.1 + 0.5 - 0.55 = 0.05$



[a] Find $P(A\cap B)$.
0.05

[b] Find $P(A\cup B)$.
0.55

[c] Find $P(A'\cap B)$.
0.45

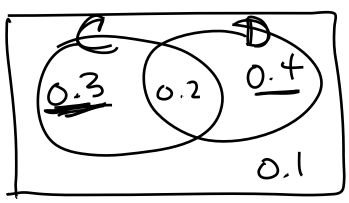
[d] Are events A and B independent? Explain how you know.

$\underline{P(A)\cdot P(B)} = (0.1)(0.5) = 0.05 = \underline{P(A\cap B)}$

A and B are independent

[2] For events C and D it is known that $P(C\cap D')=0.3$, $P(C'\cap D)=0.4$, and $\underbrace{P(C\cup D)=0.9}$.

so, $P(C'\cap D') = 0.1$



$P(C\cap D) = 1-0.3-0.4-0.1 = 0.2$

[a] Find $P(C\cap D)$
0.2

[b] Find $P(C'\cap D')$
0.1

[c] Find $P(C)$
0.5

[d] ~~[e]~~ Are C and D independent events? Explain how you know.

$P(C)\cdot P(D) = (0.5)(0.6) = 0.3 \neq P(C\cap D) = 0.2$

C and D are not independent.

[3] For events E and F it is known that $P(E)=0.20$, $P(E\cup F)=0.70$, and $P(E\cap F)=0.10$. Find $P(F)$

$P(E\cup F) = P(E)+P(F)-P(E\cap F)$

$0.7 = 0.20 + P(F) - 0.1 \Longrightarrow P(F) = 0.6$

[4] A fair coin is tossed and a fair die is rolled.

[a] Find P(3 or heads) $= P(3)+P(H)-P(3\cap H)$

$= \frac{2}{12} + \frac{6}{12} - \frac{1}{12} = \frac{7}{12}$

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

[b] Find P(3 and heads) $= \frac{1}{12}$

[c] Find P(larger than 2 and tails) $= \frac{4}{12} = \frac{1}{3}$

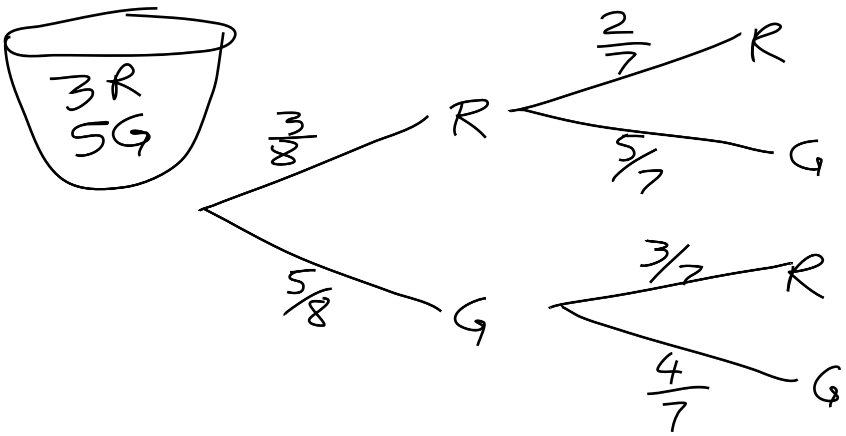
[d] Find P(larger than 2 or tails) $= \frac{10}{12} = \frac{5}{6}$

3e) Find the probability that the 1st marble is red given that the 2nd one is green.

$P(1^{st}\text{ red} \cap 2^{nd}\text{ green}) \quad \frac{15}{56}$

$P(1^{st} \text{ red} \mid 2^{nd} \text{ green}) = \frac{\quad}{P(2^{nd} \text{ green})} = \frac{\quad}{\frac{15}{56} + \frac{5}{14}} = \frac{3}{7}$

[5] A bowl contains 3 red marbles and 5 green marbles. Two marbles are drawn without replacement.
[a] Draw a probability tree to illustrate this experiment.



(d) $P(2^{nd} R \mid 1^{st} G)$

$= \frac{P(2^{nd} R \cap 1^{st} G)}{P(1^{st} G)}$

$= \frac{\frac{15}{56}}{\frac{5}{8}}$

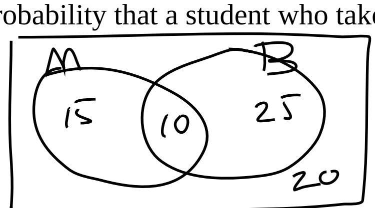
$= \frac{3}{7}$

[b] Find P(two red marbles) $\frac{3}{\cancel{8}_4} \cdot \frac{\cancel{2}}{7} = \frac{3}{28}$

[c] Find P(at least one red marble) $= 1 - P(G, G) = 1 - \frac{5}{\cancel{8}_2} \cdot \frac{\cancel{4}}{7} = 1 - \frac{5}{14} = \frac{9}{14}$

[d] Find the probability that the second marble is red given that the first one is green.

[6] In a certain school, there are 70 IB students. 25 take HL Math, 35 take HL Bio and 20 take neither of those. Find the probability that a student who takes HL Bio also takes HL Math.



$70 - 20 = 50$
$25 + 35 = 60$

$P(M \mid B) = \frac{P(M \cap B)}{P(B)} = \frac{10/\cancel{70}}{35/\cancel{70}} = \frac{2}{7}$

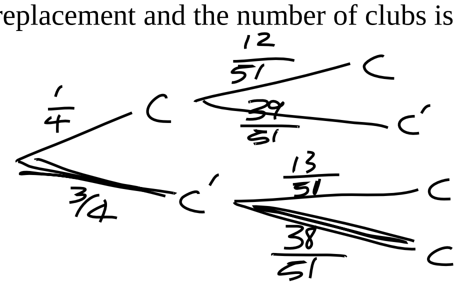
[7] For two events A and B, it is known that $P(A \mid B) = 0.4$ and $P(A \cap B) = 0.2$. Find $P(B)$.

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$0.4 = \frac{0.2}{P(B)} \rightarrow P(B) = 0.5$

[8] Two cards are drawn from a standard 52-card deck without replacement and the number of clubs is counted. (There are 13 clubs in the deck.)
[a] Find the probability that no clubs are drawn.

$\frac{3}{4} \cdot \frac{38}{51} = \frac{19}{34}$



[b] Find the probability that at least one club is drawn.

$1 - P(\text{no clubs}) = 1 - \frac{19}{34} = \frac{15}{34}$

Test on Friday