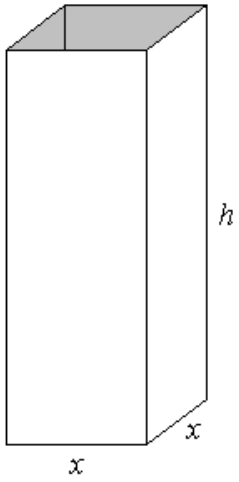
Set 019 Optimization

[1] An open-top, rectangular box with a square bottom is to be built with a volume of 2 ft^3.



[a] Write an expression for $A(x,h)$, the surface area of the box in terms of *x* and *h*.

[b] Write an equation relating *x* and *h* and solve it for *h*.

[c] Write an expression for $A(x)$.

[d] Determine the value of *x* which yields the least possible surface area for such a box.

[2] An open-top, rectangular box with a square bottom is to be built with a surface area of 5 ft^2.

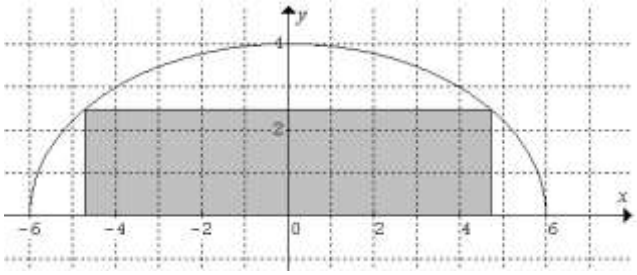
[a] Write an expression for $V(x,h)$, the volume of the box in terms of *x* and *h*.

[b] Write an equation relating *x* and *h* and solve it for *h*.

[c] Write an expression for $V(x)$.

[d] Determine the value of *x* which yields the greatest possible volume for such a box.

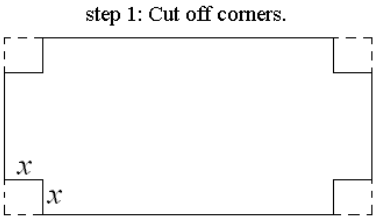
[3] A rectangle has its base on the *x*-axis and its upper vertices on the curve $\frac{x^2}{36}+\frac{y^2}{16}=1$, as shown in the diagram.



[a] Write an expression for $A(x)$, the area of the rectangle in terms of *x*, the lower, right-hand vertex.

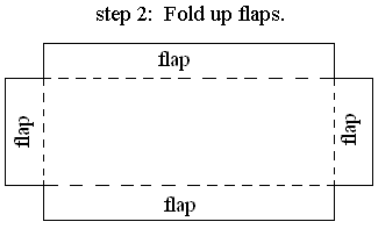
[b] Determine the value of *x* which yields the greatest possible area for such a rectangle and then state the greatest possible area.

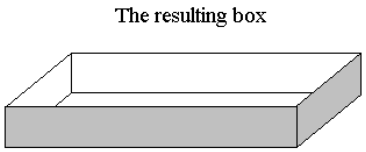
[4] An open-top box is to be constructed by cutting square corners from a rectangular piece of cardboard and then folding up the resulting flaps. The rectangular piece of cardboard is 10" by 16". Let the length of the cuts be *x* inches.

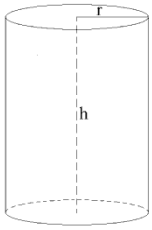


[a] Write a function for the volume of the box in terms of *x*.

[b] Determine the value of *x* which yields the greatest possible volume for such a box.

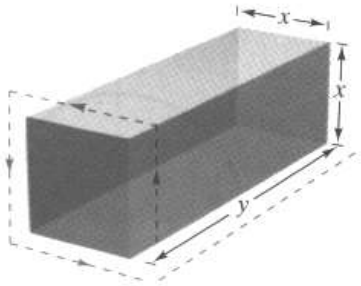




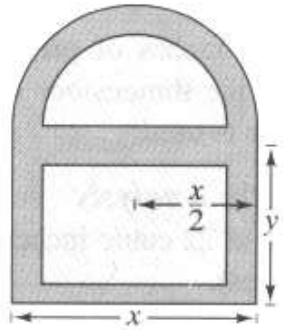


[5] A right circular cylinder is to be built to hold 16π in^3. Find the dimensions of the cylinder with the smallest possible surface area.

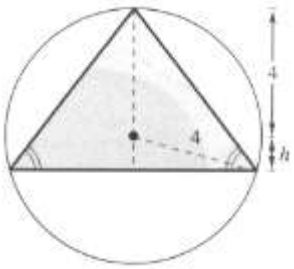
[6] Find dimensions of the largest right circular cylinder that can be inscribed in a sphere of radius *R*.



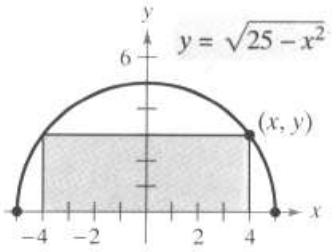
[7] A rectangular package is allowed by the shipping company to have a maximum combined length and girth of 108 inches. (The girth of the package is the perimeter of the square cross-section.) Find the dimensions of the package with the largest possible volume



[8] A Norman window is constructed by adjoining a semicircular window on top of a rectangular window. Find the dimensions of the Norman window with the largest possible area if the outer perimeter is a total of 16 feet.



[9] Find the dimensions of the largest isosceles triangle that can be inscribed in a circle of radius 4.



[10] A rectangle is bounded below by the x-axis and above by the semicircle described by $y = \sqrt{25 - x^2}$. Find the largest such rectangle.